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Car accidents determined by stopped cars and traffic flow

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Abstract

The product of traffic flow and the fraction of stopped cars is proposed to determine the probability P_{ac} for car accidents in the Fukui–Ishibashi model by analysing the necessary conditions of the occurrence of car accidents. Qualitative and quantitative characteristics of the probability P_{ac} can well be explained. A strategy for avoiding car accidents is suggested.

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1. Introduction

Recently, cellular automata (CA) models for traffic problems have been extensively studied [1, 2]. Within the framework of the CA models, the complexities of nonlinear character in traffic problems can be prescribed and clear physical picture can also be provided [3]. More importantly, the advantages of these models also show that complicated features observed under various traffic conditions such as road blocks and hindrances [4], two-level crossings [5], highway junctions [6], etc can be well described. From the viewpoint of statistical physics, on the other hand, car traffic represents a far-from-equilibrium driven system whose features have far-reaching consequences. The analytical study may lead to a profound understanding of some features of the system or may provide schemes for optimizing the capacity of the existing traffic network.

More recently, theoretical and numerical results for car accidents in the framework of the CA models have been reported. With the help of the conditions for the occurrence of car accidents first proposed by Boccara *et al* [7], analytical expressions for car accidents have been provided in the NS model with $V_{max} = 1$ in the case of $p \neq 0$ [8], and in the Fukui–Ishibashi (FI) model without considering the stochastic braking [9], respectively. However, the characteristics of the probability of car accidents in the model with $V_{max} > 1$ in the presence of stochastic braking have not been understood yet, although computer simulations of the probability for car accidents to occur in the NS model with $V_{max} > 1$ in the case of $p \neq 0$

have been offered [10]. In fact, car accidents have always been of concern, because a driver may likely come across accidents in his or her lifetime and the financial damage from traffic due to accidents is huge every year. Thus, analytical studies for the occurrence of car accidents in the case of $V_{\max} > 1$ and how to effectively avoid the occurrence of car accidents are highly desirable.

In this paper, we mainly study the probability for car accidents to occur in the FI model. By analysing the necessary conditions of the occurrence of accidents, we find a universal approach to study car accidents. The product of the traffic flow and the fraction of stopped cars will be proposed to determine the value of the probability for the occurrence of car accidents. These analytical results can predict the probability for car accidents to occur, not only in the deterministic FI model, but also in the stochastic FI (sFI) model with $V_{\max} > 1$. Utilizing these analytic results, we will suggest a scheme to avoid the occurrence of car accidents.

2. Conditions for car accidents to occur

We consider the cellular automata model introduced by Fukui and Ishibashi [11] to describe single-lane highway traffic. The model consists of a one-dimensional array of L cells with periodic boundary conditions. Every cell can be either empty or occupied by a car with velocity $v = 0, 1, 2, \ldots, V_{\max}$, where $V_{\max}$ is the speed limit. Let d_E denote the number of empty cells in front of a car and N the number of cars on the road. The following steps for all cars are performed in parallel. The first rule is acceleration: if the speed of a car is lower than $V_{\max}$, then it equals the value of the speed limit. The second rule is deceleration due to other cars: if the speed is larger than d_E, then it is reduced to d_E. The third rule is randomization: if the speed of a moving car is the speed limit, then it is decreased by one unit with a braking probability p. And the fourth rule is that each car is moved forward according to its new speed determined by the above three rules.

The FI model is one of the basic CA models for describing traffic flow. It is different from the NS model proposed by Nagel and Schreckenberg [12], in the following two aspects: (1) the increase of speed of the cars is not gradual and (2) the stochastic delay applies only to high-speed cars. Obviously, the FI model reduces to the NS model if $V_{\max} = 1$.

Because keeping a safe distance is given in the second rule of update, car accidents do not happen in the basic FI model. However, in real traffic, car accidents often occur if the conditions for safe driving are not satisfied. More precisely, if a car ahead is moving, expecting it to be moving at the next time step, a careless driver has a tendency to drive as fast as possible and increases the safe velocity given in the second rule of update by one unit with a probability p'. At the next time step, it will arrive at the position of the moving car ahead. If the moving car ahead is suddenly stopped, collision between the two cars happens. Let $x(i, t)$ and $v(i, t)$ denote the position and velocity of the ith car at time t, respectively. The three necessary conditions for determining the occurrence of car accidents which have been proposed by Boccara *et al* read as follows. The first condition is $d_E \leqslant V_{\max}$, which means that the position of the car ahead can be reached by the car at the next time. The second condition is $v(i+1, t) > 0$. The last condition is $v(i+1, t+1) = 0$, which represents that the moving car $i+1$ is suddenly stopped at time $t+1$. These conditions concern states of two neighbourhood cars at different time steps; therefore they are correlative, both spatial and temporal. When the three conditions are simultaneously satisfied, the dangerous situation of a car accident occurring appears. If the careless driver of the ith car increases speed by one unit with probability p', a car accident will occur. Obviously, the occurrence of car accidents is proportional to the occurrence of dangerous situations. The proportional constant is p'. Usually, the probability per car per time step for car accidents to occur is denoted by P_{ac}. Obviously, P_{ac} is proportional to the probability p'. As p' is constant, the accident probability P_{ac} can be scaled by p'. In computer simulations, car accidents do not really happen. These dangerous situations are calculated and considered as a signal of the occurrence of car accidents.

In the FI model, there are three basic parameters: the average car density ρ, the speed limit $V_{\max}$ and the stochastic delay probability p. The probability for the occurrence of car accidents will be a function of these parameters.

Due to the third condition for car accidents, the probability P_{ac} is directly related to the stopped cars. In the FI model, when the car density ρ is less than the critical density $\rho_c = \frac{1}{(1+V_{\max})}$ at which a phase transition from the freely moving phase to the jamming phase occurs, the average distance between two consecutive cars is larger than $V_{\max}$. In that case, all cars can move forward in the case of $V_{\max} \geqslant 2$, therefore no stopped car exists and no car accident occurs. However, when $\rho > \rho_c$, some cars become stopped and collisions between cars happen. Thus, the probability for the occurrence of car accidents exists only when the car density is larger than the critical density.

However, the stopped cars are not a unique ingredient for determining car accidents. From the existing results [7–10], we know that the probability is a nonlinear function of the car density, while the fraction of the stopped cars approximately linearly increases with increasing car density when the density ρ is larger than the critical density ρ_c. Thus, besides the stopped cars, there are other elements that can influence the probability for car accidents. In the following section, to understand car accidents, we will obtain exact expressions by analysing the conditions of the occurrence of car accidents and compare them with numerical results. In the process of simulations, numerical data are obtained from an average over 5000 time steps and 50 ensembles in a system of $L = 5 \times 10^3$.

3. Theoretical analysis

We first study the probability for car accidents to occur in the deterministic FI models. The stochastic braking is neglected, i.e., $p = 0$. Assume that the average velocity of the cars on the road is denoted by $\langle V\rangle$, and the mean traffic flow by $\langle J\rangle$. According to the definition of the mean traffic flow, $\langle J\rangle$ means that there are on average $\langle J\rangle$ cars which pass through one cell per time step. If a cell is occupied by a suddenly stopped car, the probability for the occurrence of car accidents per time step at the cell is $p'\langle J\rangle$. Obviously, if the number of stopped cars on the road is N_0, the probability for car accidents to occur per time step on the road equals $p'\langle J\rangle N_0$. Thus, the probability P_{ac} can be written as

$$\begin{aligned} P_{\mathrm{ac}} &= \frac{p'\langle J\rangle N_0}{N} \\ &= p' n_0 \langle J\rangle \\ &= p' c_0 \langle V\rangle \end{aligned} \tag{1}$$

where $n_0 = \frac{N_0}{N}$ is the fraction of stopped cars and $c_0 = \frac{N_0}{L}$ is the density of the car with the stationary state. Therefore, P_{ac} is proportional to the product of the mean traffic flow and the fraction of the stopped cars, also proportional to the product of the mean velocity and the density of the stopped cars.

The correctness of equation (1) is easily verified. In fact, in the FI model without considering car accidents, a graph of mean traffic flow as a function of car density, which is called a fundamental diagram, is extensively studied. Earlier, a site-oriented mean-field (SOMF) theory and a car-oriented mean-field (COMF) theory [1] were developed for the NS model, and exact results have been obtained if $V_{\max} = 1$. Recently, Wang *et al* developed the above theories for obtaining the average traffic flow for arbitrary $V_{\max}$ and the braking probability in the FI model, by starting from the microscopic update rule of a Boolean variable related to the occupancy defined on each site [13], and by introducing the concepts of intercar spacings [14].

Exact results in the deterministic FI model have also been proposed [11, 15]. With the known results, above the critical density, the fraction of stopped cars and the mean velocity can be respectively read as (see [11])

$$n_0 = 1 - d \tag{2}$$

$$\langle V \rangle = \sum_{i=1}^{V_{\max}} d^i = \frac{1-\rho}{\rho} \tag{3}$$

where d is the probability for a site being empty. With the help of equations (2) and (3), the mean velocity can be rewritten as

$$\langle V \rangle = d \sum_{i=0}^{V_{\max}-1} d^i = (1-n_0)\left(\sum_{i=0}^{V_{\max}-1} d^i\right) = \left(\sum_{i=1}^{V_{\max}} n_i\right)\left(\sum_{i=0}^{V_{\max}-1} d^i\right) \tag{4}$$

where n_i is the fraction of cars with velocity i, and $1 - n_0$ means the total fraction of cars with positive velocity which equals $\sum_{i=1}^{V_{\max}} n_i$. Substituting equation (4) into equation (1), we have

$$P_{\mathrm{ac}} = \frac{p'}{\rho}\left(\rho \sum_{i=1}^{V_{\max}} n_i\right)(\rho n_0)\left(\sum_{i=0}^{V_{\max}-1} d^i\right) \tag{5}$$

where the relation, $c_0 = \rho n_0$, is utilized. Obviously, equation (5) is the analytical expression obtained from the mean-field theory according to the three constrained conditions of the occurrence of car accidents first proposed by Boccara *et al* [7, 9]. The term $\left(\rho \sum_{i=1}^{V_{\max}} n_i\right)$ in equation (5) is the probability of a cell being occupied by a moving car, the term ρn_0 is the probability of a cell being occupied by a stopped car, and the last term is the probability of a careless driver arriving at the cell occupied by the car ahead during one time step. Thus, equation (1) is equivalent to the three constrained conditions for a car accident in the deterministic FI model.

Utilizing equation (1), qualitative features of the probability P_{ac} can be analysed. Below the critical density, all cars move with the maximum velocity, and no stopped car exists on the road. However, above the critical density, the value of $\langle J \rangle$ linearly decreases from the maximum value to zero with the increase of ρ, and in contrast, the value of n_0 increases from zero to one. As described in equation (1), the probability P_{ac} is zero below ρ_c because of no stopped cars. Above ρ_c, the probability P_{ac} which is determined by the product of $\langle J \rangle$ and n_0 becomes a non-monotonic function of the car density ρ (see figure 1).

As the speed limit $V_{\max}$ increases, the value of $\langle J \rangle$ increases in the low-density region, then leading to the increase of the probability P_{ac}. In the high-density region, where the fraction of the stopped cars becomes a larger value and independent of the speed limit, the traffic flow is irrespective of $V_{\max}$, thus a scaling relation of the probability P_{ac} is expected as the speed limit increases (see figure 1).

Figures 2 and 3 show the dependence of the probability P_{ac} on the fraction of the stopped cars n_0 and the traffic flow $\langle J \rangle$, respectively. As shown in figures 2 and 3, the probability P_{ac} is a nonlinear function of n_0 or $\langle J \rangle$, and linear behaviour between P_{ac} and n_0 or $\langle J \rangle$ is also observed in two limit density regions. The features of P_{ac} can be explained as follows. Below the critical density, there is no stopped car and therefore no accidents, whether the traffic flow exists or not. Near the critical density, n_0 assumes a small value, while $\langle J \rangle$ approaches

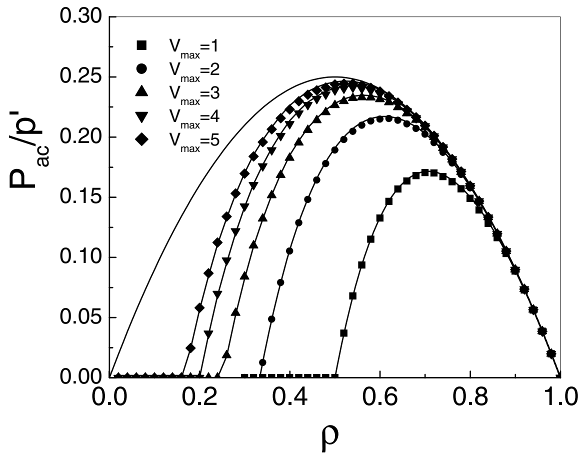

Figure 1. Probability P_{ac} (scaled by p') as a function of car density ρ in the deterministic FI model. Solid lines correspond to the analytical results and symbol data are obtained from numerical simulations.

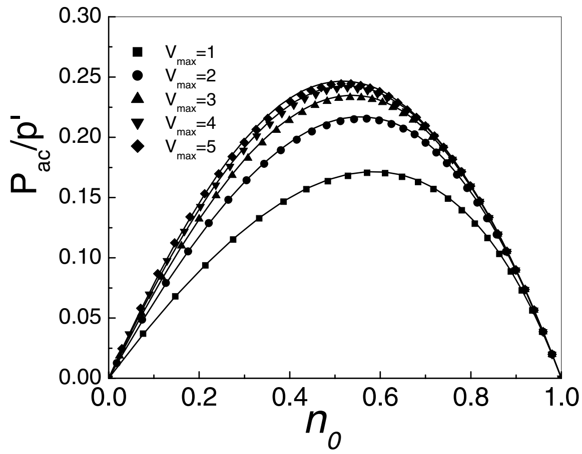

Figure 2. The relation of the probability P_{ac} (scaled by p') to the fraction of stopped cars n_0 in the deterministic model. Solid lines correspond to analytical results and symbol data are obtained from simulations.

a maximal value, thus the value of P_{ac} is proportional to n_0 (see figure 2). In the limiting case $\rho \to 1$, where the fraction of the stopped cars tends to one, the traffic flow becomes a stop-and-go wave, so the value of P_{ac} is proportional to $\langle J \rangle$ (see figure 3). Because the increase of n_0 leads to the decrease of $\langle J \rangle$, and vice versa, the value of P_{ac} linearly decreases with the increase of $\langle J \rangle$ in the high traffic flow region and linearly decreases with the increase of n_0 in the high-density region (see figures 2 and 3).

In figure 2, if the value of n_0 is fixed in a moderate region, the increase of the speed limit enhances the probability P_{ac} owing to the increase of the traffic flow. In figure 3, if $\langle J \rangle$ is larger, the increase of the speed limit also raises the probability P_{ac} because of the increase of n_0. Thus, qualitative characteristics of the probability P_{ac} can be explained very well.

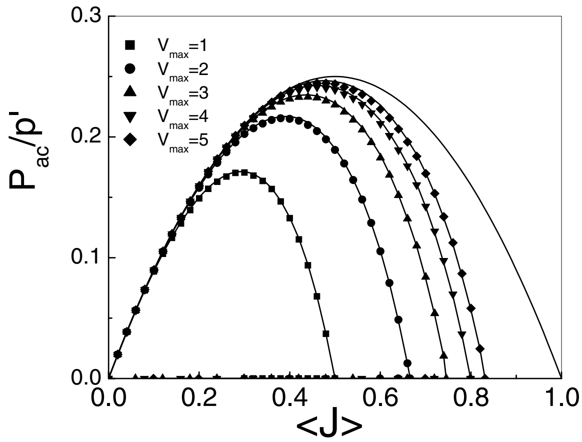

Figure 3. The relation of the probability P_{ac} (scaled by p') to the traffic flow $\langle J\rangle$ in the deterministic FI model. Solid lines correspond to analytical results and symbol data are obtained from simulations.

Quantitatively, we can easily obtain the exact expressions for the fraction of the stopped cars n_0 by using equations (2) and (3). Above the critical density, n_0 can be respectively written as

$$n_0 = \begin{cases} 2 - \frac{1}{\rho} & \text{for} \quad V_{\max} = 1 \\ \frac{3}{2} - \sqrt{\frac{1}{\rho} - \frac{3}{4}} & \text{for} \quad V_{\max} = 2 \\ \rho & \text{for} \quad V_{\max} \to \infty \end{cases} \tag{6}$$

and the traffic flow $\langle J\rangle$ equals $1 - \rho$ for $\rho > \rho_c$. With the help of equation (1), the exact results for the probability P_{ac} can be obtained by

$$P_{\mathrm{ac}} = p' \begin{cases} \left(2 - \frac{1}{\rho}\right)(1-\rho) & \text{for} \quad V_{\max} = 1 \\ \left(\frac{3}{2} - \sqrt{\frac{1}{\rho} - \frac{3}{4}}\right)(1-\rho) & \text{for} \quad V_{\max} = 2 \\ \rho(1-\rho) & \text{for} \quad V_{\max} \to \infty \end{cases} \tag{7}$$

which corresponds to the results obtained from a mean-field theory [9].

The relations of the accident probability to the fraction of the stopped cars and the traffic flow are also obtained from equations (1) and (6), respectively:

$$P_{\mathrm{ac}} = p' \begin{cases} \frac{n_0(1-n_0)}{2-n_0} & \text{for} \quad V_{\max} = 1 \\ \frac{n_0(n_0-1)(n_0-2)}{n_0^2-3n_0+3} & \text{for} \quad V_{\max} = 2 \\ n_0(1-n_0) & \text{for} \quad V_{\max} \to \infty \end{cases} \tag{8}$$

$$P_{\mathrm{ac}} = p' \begin{cases} \frac{\langle J\rangle(1-2\langle J\rangle)}{1-\langle J\rangle} & \text{for} \quad V_{\max} = 1 \\ \langle J\rangle\left(\frac{3}{2} - \sqrt{\frac{1+3\langle J\rangle}{4(1-\langle J\rangle)}}\right) & \text{for} \quad V_{\max} = 2 \\ \langle J\rangle(1-\langle J\rangle) & \text{for} \quad V_{\max} \to \infty. \end{cases} \tag{9}$$

The numerical results are in good agreement with expressions (7) and (8), as shown in figures 2 and 3.

Next, we consider the case of the stochastic delay. Unlike the NS model, the stochastic delay in the sFI model applies only to the cars with maximal velocity. Thus, in the presence

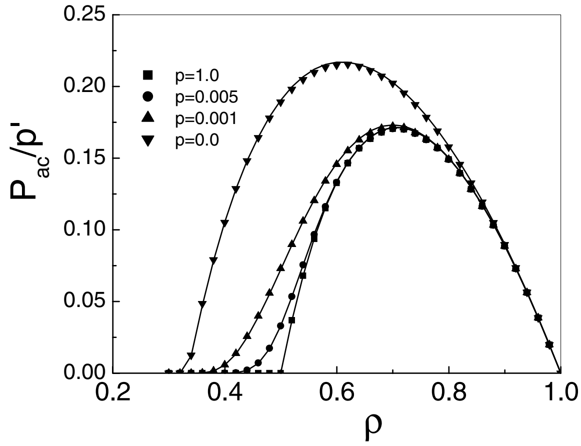

Figure 4. The probability P_{ac} (scaled by p') as a function of the car density ρ in the sFI model with $V_{max} = 2$. Solid lines correspond to analytical results and symbol data are obtained from simulations.

of braking, no correlation exists between the traffic flow and stopped cars. An exception is the case of $V_{max} = 1$, where correlations between the traffic flow and the stopped cars are induced by the braking. Thus, equation (1) can give exact results of the probability P_{ac} not only in the deterministic case, but it can also prescribe the probability for car accidents to occur in the case of the stochastic delay.

Figure 4 shows the dependence of the probability P_{ac} on the car density ρ in the sFI model with $V_{max} = 2$. Owing to the lack of explicit expressions for n_0, theoretical results of the probability P_{ac} are obtained by calculating the product of $\langle J \rangle$ and n_0. In figure 4, square, circle, up triangle and down triangle data are results from computer simulations in the case of $p = 0, 0.001, 0.005$ and 1, respectively, and our analytical results are indicated by solid lines. Comparison of our prediction for P_{ac} with computer simulations shows excellent agreement.

As shown in figure 4, the relation of the probability P_{ac} to the braking in the FI model exhibits some unique features which can be well explained with the help of equation (1). In figure 4, if the value of p is very small, the probability P_{ac} is suppressed in the low-density region, and does not vary in the high-density region with increasing braking probability. However, if the value of p is larger, the probability P_{ac} is the same as that corresponding to the totally delayed model with $V_{max} - 1$ and independent of p.

To well explain the character of the probability P_{ac}, we calculate the relation of n_0 to ρ, which is shown in figure 5. When the value of p is very small (for example, near 0.001 in the case $V_{max} = 2$), the slight increase of p enhances the density for the onset of n_0 markedly and reduces the value of n_0 in the low-density region, due to the stochastic delay applying only to the cars at the speed limit. But in the high-density region where all cars move forward with speed smaller than the maximum speed, the value of n_0 is independent of p. In the opposite limit case, when the value of p is larger, most cars with maximum speed are delayed, therefore the distribution of n_0 behaves in the same way for different delay probabilities and almost corresponds to the totally delayed model with $V_{max} - 1$. For the sFI model, the traffic flow $\langle J \rangle$ which can result in car accidents varies only in between two critical densities, corresponding to the deterministic model with $p = 0$ and $p = 1$ as the car density ρ increases [11], respectively. Therefore, according to equation (1), when p is smaller the increasing of p leads to a lower value of P_{ac} in the low-density region and causes the onset of P_{ac} to move towards the

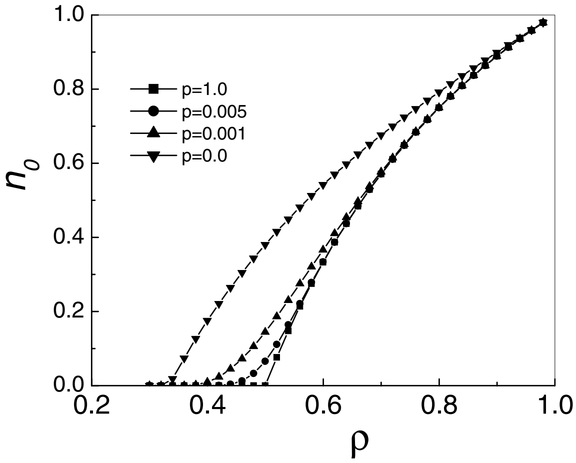

Figure 5. The relation of the fraction of stopped cars n_0 to the car density ρ in sFI model with $V_{\max} = 2$.

high-density region, but does not change the value of P_{ac} in the high-density region; when p is larger, the distributions of P_{ac} with different values of p shrink into one curve corresponding to the totally delayed model with $V_{\max} - 1$.

Now, we discuss strategies for avoiding car accidents. In the basic FI model, there are three basic parameters: the car density $\rho = \frac{N}{L}$, the braking probability p and the speed limit $V_{\max}$. The probability P_{ac} which is a function of the above three parameters has been studied previously [7–10]. However, in real traffic systems, only traffic flow and the stopped cars can be measured. Therefore expression (1) for the probability P_{ac} is of practical relevance. To avoid the occurrence of car accidents, we usually adopt a scheme that the traffic flow is decreased by traffic lights or that the stopped cars on the road are removed by police cars. However, the decrease of traffic flow (the stopped cars) may lead to an increase in the fraction of stopped cars (traffic flow) in some real traffic systems, i.e., the number of cars in the system remains unchanged. According to expression (1), the decrease of traffic flow or the stopped cars cannot suppress, but instead enhances the probability of occurrence of car accidents in a certain situation. Therefore, to effectively avoid the occurrence of car accidents, both traffic flow and stopped cars must be considered simultaneously.

4. Conclusion

We propose a phenomenological method to study car accidents in the FI model. The probability P_{ac} per car per time step for an accident to occur is proportional to the product of n_0 and $\langle J \rangle$. The conclusion can be proved to be equivalent to the three constrained conditions for the occurrence of car accidents first proposed by Boccara *et al* in the deterministic FI model. The analytical results of the probability P_{ac} in the case of stochastic delays show excellent agreement with simulations. Therefore, a phenomenological mean-field theory for calculating the probability P_{ac} becomes very effective in the FI model. Utilizing these analytic results, we suggest that both traffic flow and the fraction of the stopped cars on the road must be controlled simultaneously to suppress the occurrence of car accidents.

Utilizing equation (1), features of the probability P_{ac} with the changes of the stochastic delay have been well explained. In the sFI model, when the value of p is very small, P_{ac}

decreases with the increase of p in the low-density region and is independent of the variety of p in the high-density region. When the value of p is larger, the distributions of P_{ac} corresponding to different values of p fall into one curve. The present approach provides an alternative way to study the problems of car accidents analytically. In principle, our approach can be extended to study other models.

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